Music, Machine, and Mathematics

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Overview

Mysterious Melody(Diana Deutsch, 1972)

Scrambled

ClickToPlay

Mysterious Melody(Diana Deutsch, 1972)

Scrambled

ClickToPlay

UnScrambled

ClickToPlay

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Scrambled

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UnScrambled

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Conclusion

Sound(Physics) + Ear(Biology) + Brain(??) = Perception Of Sound(What We Hear)!

Physics Of Sound

Sound

Sound is created by repetitive pressure change in the air. We can hear it when the repetition occurs roughly 20 to 20000 times a second.

Components Of Sound

A **Musician** describes a sustained, musical tone in terms of three quantities:

- **Pitch**
- **Loudness**
- **Timbre**

A **Physicist** would describe the same tone in terms of three quantities:

- **Frequency**
- **Intensity**
- **Overtone or Harmonic Structure**

Sound From A Piano

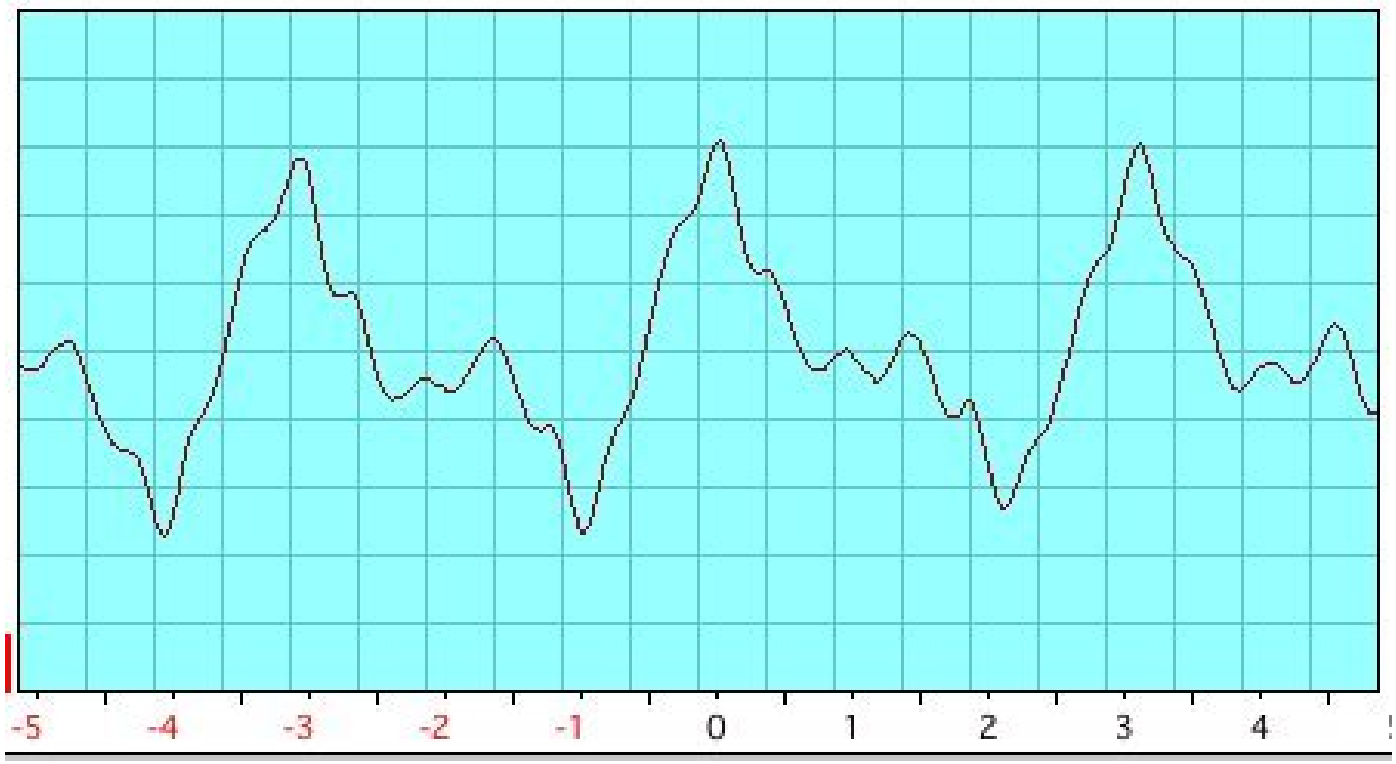

Figure: Piano (A below C4)

Frequency Chart

Pitch/Frequency

Observation

Musical tones are generated by (almost) periodic change of pressure in the air. The period(frequency) is perceived as the **Pitch** of the tone.

11000 To 20000 Hertz

How many you could hear? How old are you? ClickToPlay

Perception Of Frequency Is Not Linear

This plays tones of pitch evenly spaced: 500,1500, 2500, 3500, 4500, 6500 Hertz. ClickToPlay

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Perception Of Frequency Is Logarithmic(Base 2)

Do these sound evenly spaced? Ratio2

Loudness/Intensity

Definition

- To a physicist "Intensity=**Power/Area**".
- To a musician "how loud it sounds to his ear"(**which depends on one's perception**).

Range Of Perceivable Loudness

Just as we hear sounds which occur in a wide range of frequencies, we hear sounds in a very wide range of loudnesses. Sometimes it is important to be able to hear that someone is breathing in the same room as you. Sometimes someone will shout in your ear. The sound intensity of the shout may be more than a billion times larger than the loudness of the breathing. You have to be able to usefully perceive both.

Loundness/Intensity

Perception of loudness is Not Linear again! **The actual relation of intensity with loudness has to do with how the neurons are triggered by the intensity and duration of sound**.

Measuring Loudness

A standard way to measure loudness is **Bel** which is be definition
$1 Bel = log_{10} \frac{Intensity}{10^{-12} W/m^2}$
1 Bel is too loud! For practical purposes we use **deciBel** $= \frac{1}{10}$ **Bel**.

A Rule Of Thumb

10 TIMES INCREASE IN INTENSITY IS PERCEIVED AS A 2 TIMES INCREASE IN PERCEIVED LOUDNESS. ClickToPlay

Pitch And Loudness Are Not Independent

Tones of the same intensity, but of different frequency, perceived as being of different loundness.

HearingTestWebsite

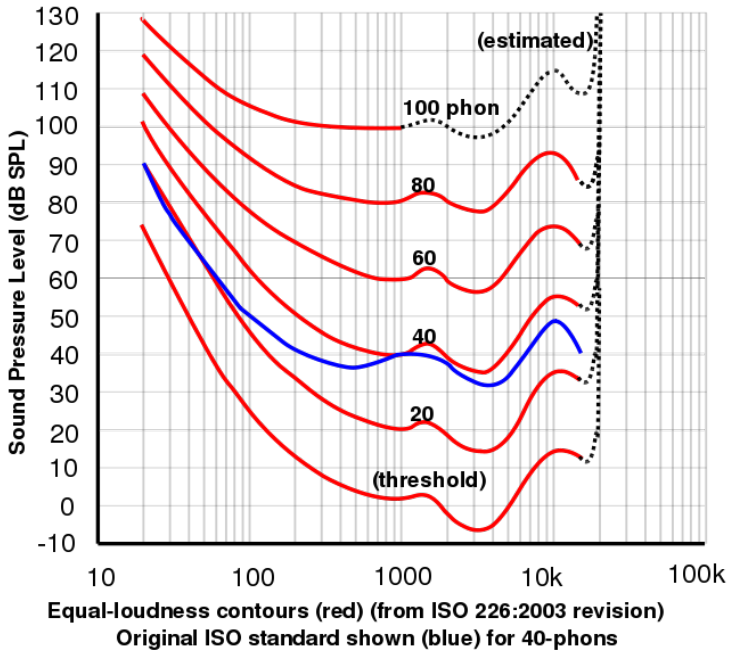
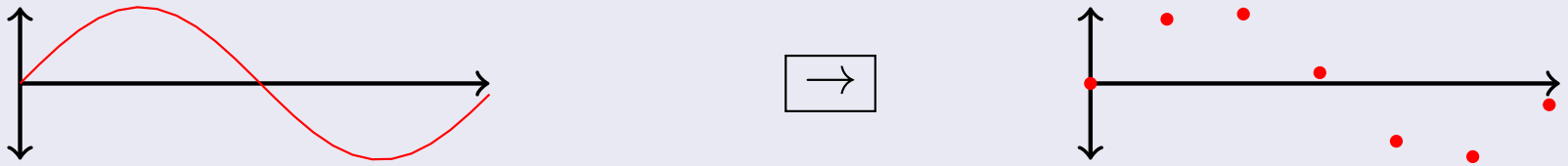


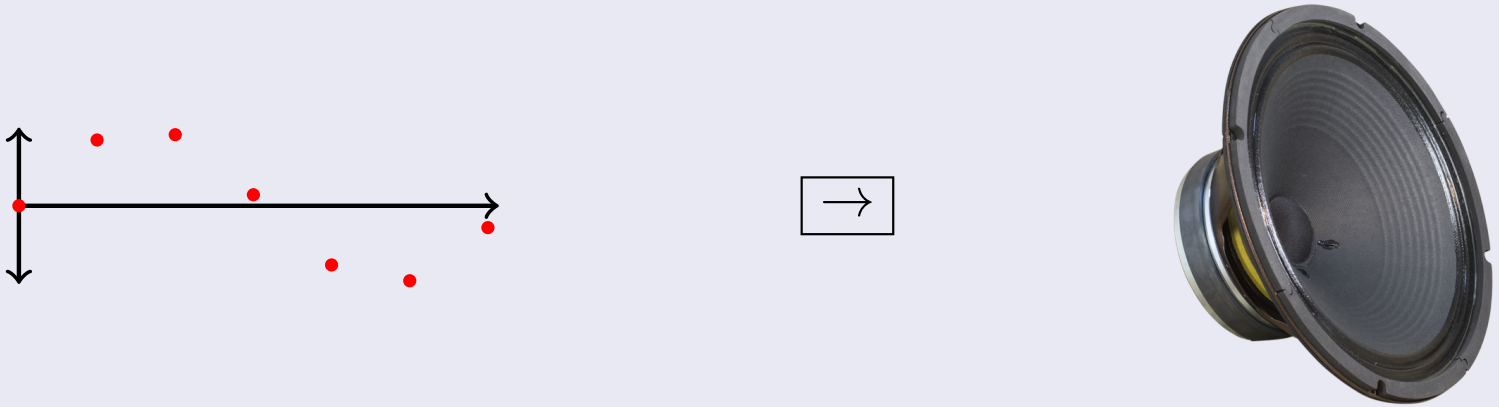
Figure: Equal Loudness Contours

Computer Generation of Sound

What is Sound on a computer?



Then to a speaker



How to choose the sampling rate

Sampling rate = discretization

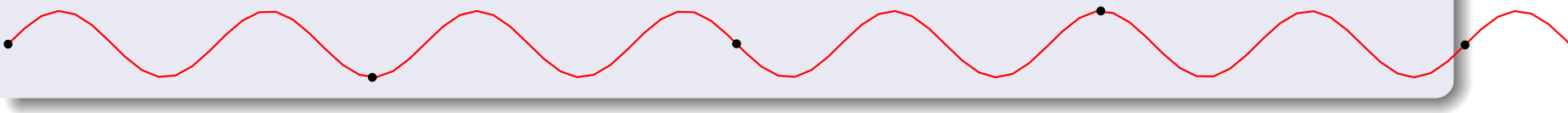
That is to say, the sampling rate has the same issues as discretization choices do in numerical mathematics. How can the necessary information be captured most efficiently?

CDs

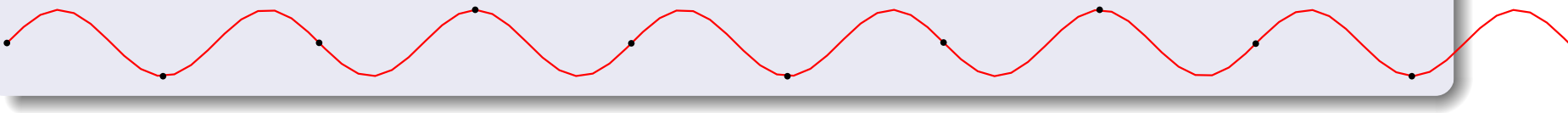
CDs use 44.1 kHz sampling. Why?

Capturing a pressure wave

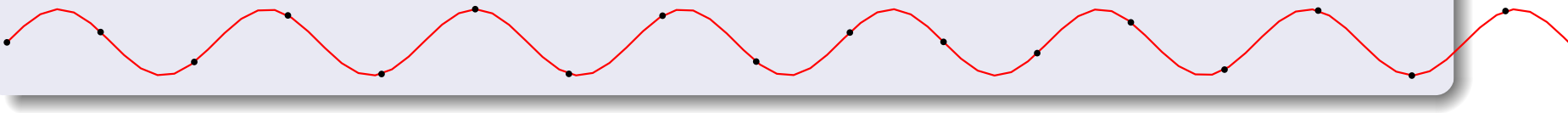
Sampling at 1.75 of the period



Sampling at 0.75 of the period



Sampling at 0.45 of the period



Nyquist

Nyquist-Shannon Theorem

If a function Q is composed of continuous periodic waves (i.e. a nice fourier series) and has a highest frequency component in hertz f_{max} then the data of a sampling rate of at least $2f_{max}$ can be used to exactly determine Q.

Pd Examples

How Can 2 Sound Waves Interact?

First, what happens when two sound waves meet?

They add.

The math of 2 wave friends

Wave_1 & Wave_2

Suppose $\text{Wave}_1 = A\cos(wt + k)$ and $\text{Wave}_2 = A\cos(w't + k')$. Then,

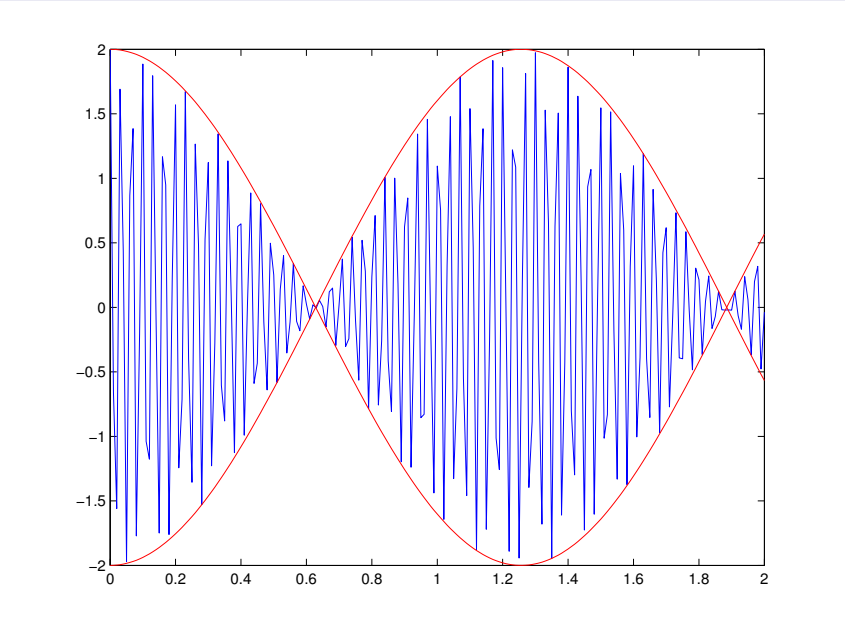
$$\begin{aligned} W_1 + W_2 &= A\cos(wt + k) + A\cos(w't + k') \\ &= 2A\cos\left(\frac{(w + w')t + (k + k')}{2}\right)\cos\left(\frac{(w - w')t + (k - k')}{2}\right) \end{aligned}$$

Aural Beating

The math of the given example

Suppose $\text{Wave}_1 = \cos(440t)$ and $\text{Wave}_2 = \cos(435t)$. Then,

$$\begin{aligned} W_1 + W_2 &= \cos(440t) + \cos(435t) \\ &= 2\cos\left(\frac{875t}{2}\right)\cos\left(\frac{5t}{2}\right) \end{aligned}$$



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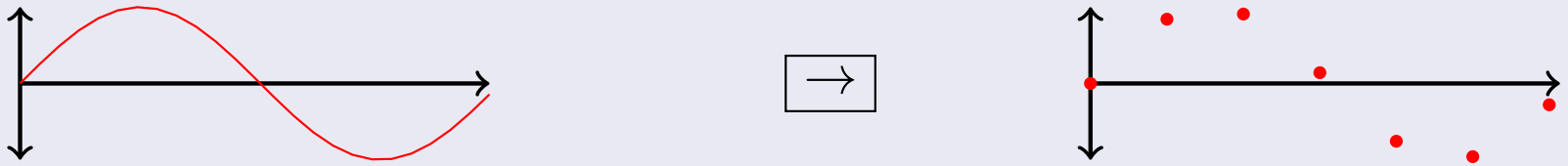

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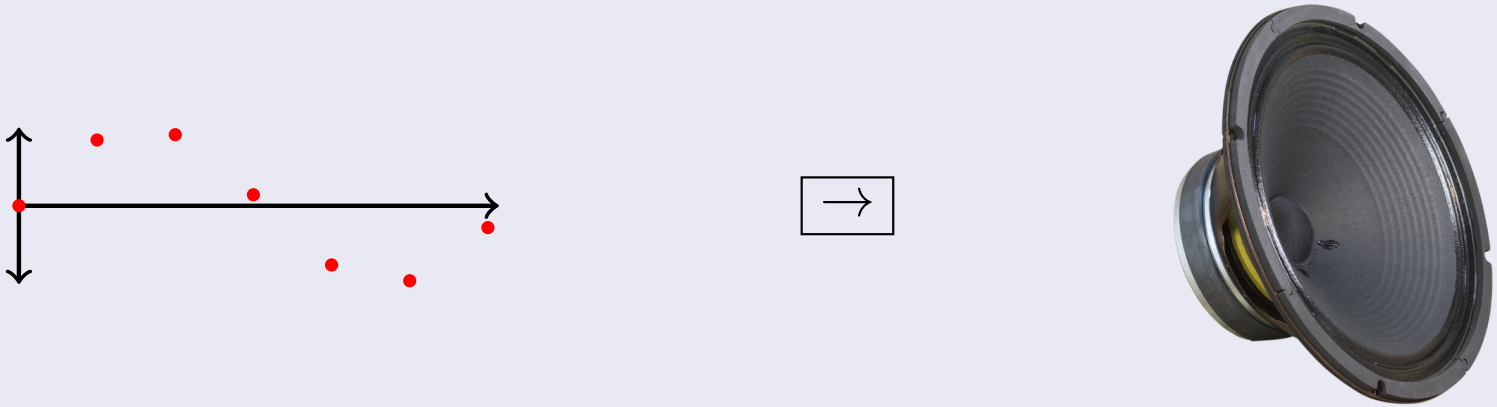
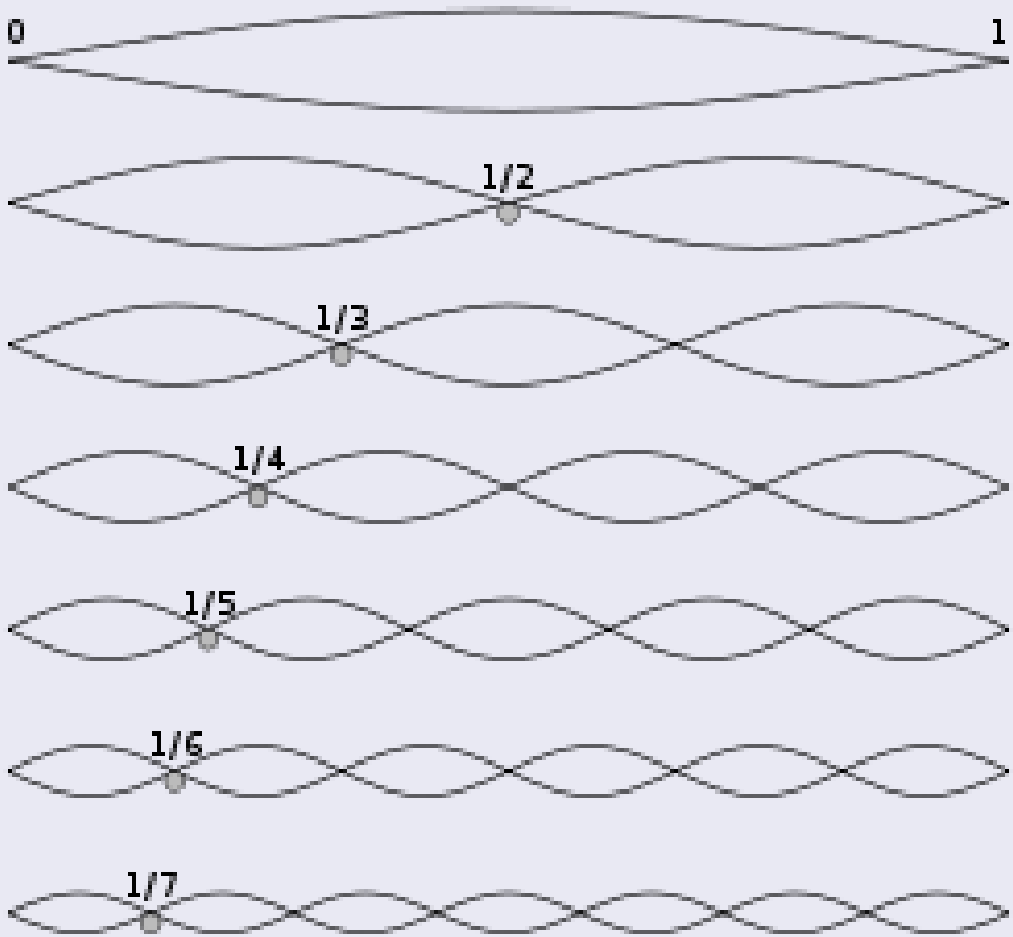


Sampling at Δt

Sampling at same rate of a higher frequency tone

Sounds from instruments

Fundamental and overtones



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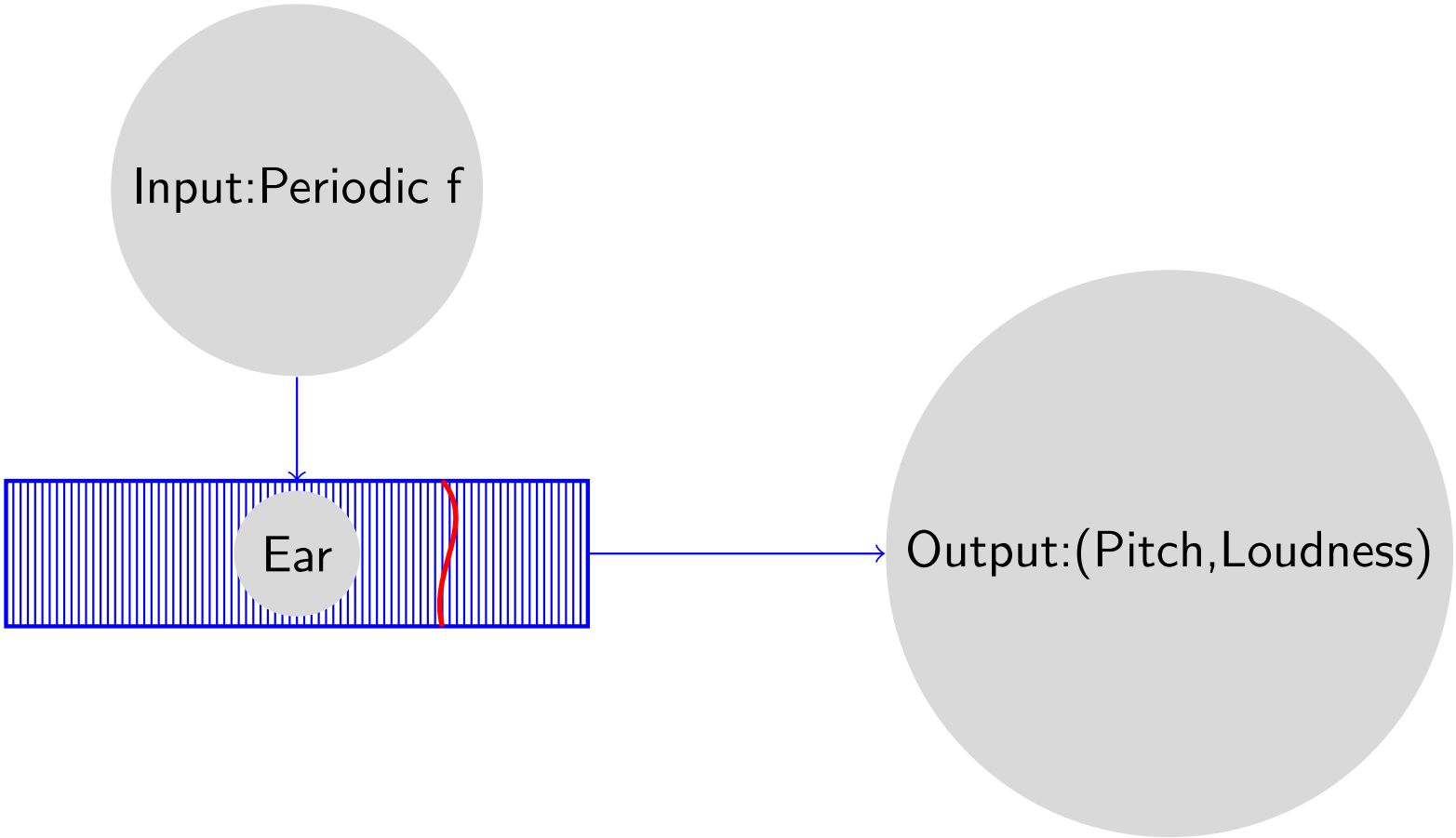
How We Hear



Some Random Sounds

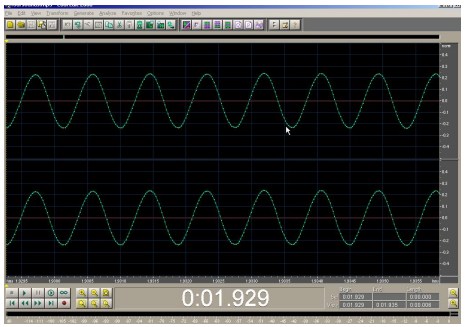


Figure: Wineglass

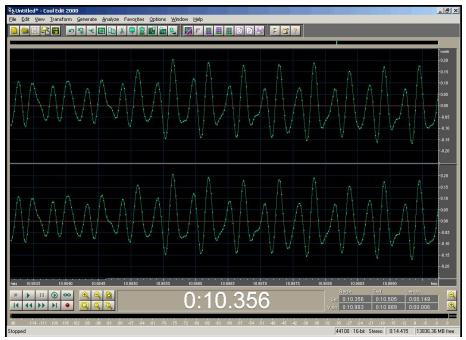


Figure: Siren

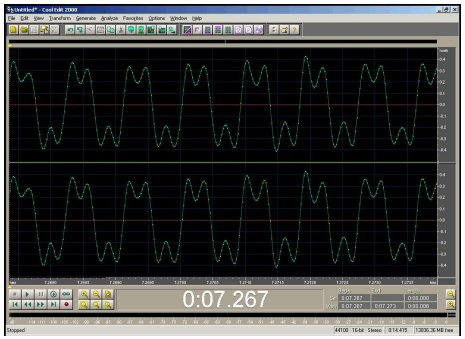


Figure: Whistle

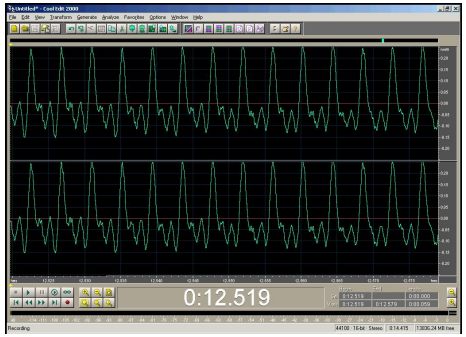


Figure: Voice

ClickToPlay

Fourier Analysis Of Sound

Good News

Superposition of sound works for our brain, the way superposition of waves works in nature!

If several things are producing sounds at once, then the pressure of the air, due to the several things, will be

$P_{air} = P_{atmospheirc} + P_{source1} + P_{source2} + \ldots$

Power Of Mathematics

As we have seen Our ears like periodic tones. And **Fourier** says, any periodic function $f(x)$, say the pressure pattern from an instrument, can be viewed as the sum of many Sine waves.

Timbre(Fourier Modes)

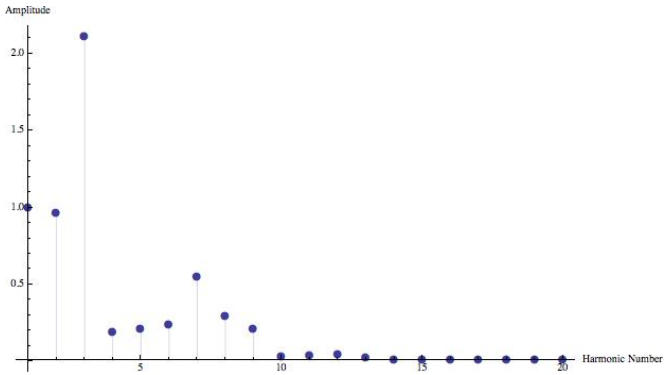


Figure: Oboe

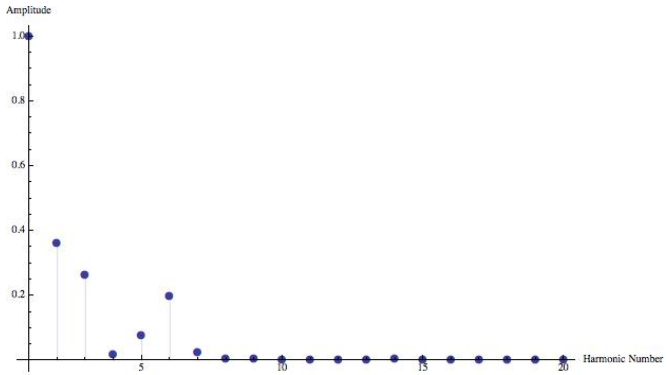


Figure: Clarinet

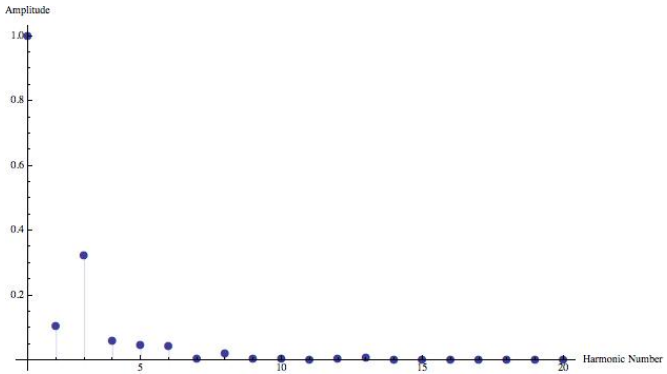


Figure: Piano

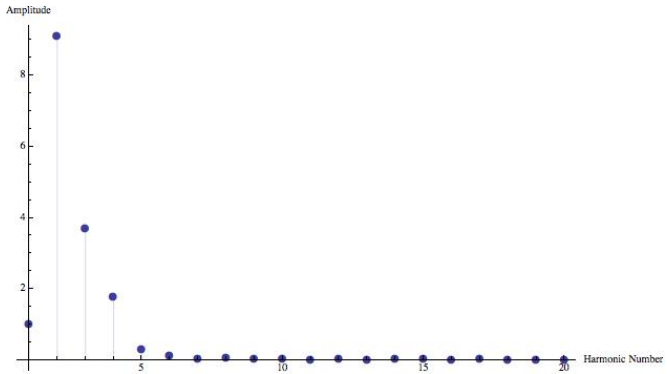


Figure: Flute

How To Produce Realistic Sounds

WaveForm Patch
Synthesizer Patch

FM Synthesis

FM Synthethis Patch

Thank You!